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Non-abelian brane worlds: the heterotic string story

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Abstract: We discuss chiral supersymmetric compactifications of the SO(32) heterotic string on Calabi-Yau manifolds equipped with direct sums of stable bundles with structure group U(n). In addition we allow for non-perturbative heterotic five-branes. These models are S-dual to type I compactifications with D9- and D5-branes, which by themselves are mirror symmetric to general intersecting D6-brane models. For the construction of concrete examples we consider elliptically fibered Calabi-Yau manifolds with SU(n) bundles given by the spectral cover construction. The U(n) bundles are obtained via twisting by line bundles. We present a four-generation Pati-Salam and a three-generation Standard-like model.

Keywords: Anomalies in Field and String Theories, Superstrings and Heterotic Strings, String Duality, Superstring Vacua.

Contents

1. Introduction

Despite continuous studies of four-dimensional string compactifications it is believed that only a very tiny subset of the vast landscape of all string vacua has by now been addressed. So far most model building attempts have focused on either the $E_8 \times E_8$ heterotic string or, more recently, on type I strings and its generalizations to type II orientifolds.

The extensively studied heterotic backgrounds include toroidal orbifolds with Wilson lines (see e.g. [1]), Calabi-Yau manifolds given by hypersurfaces on toric varieties (see e.g. [2]) and elliptically fibered Calabi-Yau manifolds equipped with SU(n) bundles (see e.g. [3 – 6]) defined via the spectral cover construction of Friedman, Morgan and Witten [7].

On the open string side a promising class is given by intersecting D6-brane models in type IIA, which have primarily been studied on toroidal orbifolds (see [8 – 11] for reviews). There have only been a few attempts to extend these constructions to more generic smooth Calabi-Yau manifolds [12, 13]. Progress has mainly been hampered by the fact that not very much is known about special lagrangian three-cycles on concrete Calabi-Yau's. Remarkably, the powerful techniques of boundary conformal field theory as applied in the framework of Gepner models allow for an algebraic construction of a plethora of such orientifolds with the Kähler moduli fixed at string scale radii [14 – 16].

Via mirror symmetry geometric intersecting D6-brane models are related to type IIB orientifolds with D9(D7) and D5(D3) branes. As B-type branes they also carry some in general non-abelian gauge bundles. S-duality [17] maps such type I models to compactifications of the SO(32) heterotic string.

A detailed study of this type of string vacua is of immediate interest. One obvious reason is, of course, the long-standing problem of implementing Standard-model like physics into fully-fletched string compactifications. In view of the recent discussion about the landscape [18], one can formulate this question as: Does the MSSM or any generalisation to be found at the LHC couple consistently to gravity such that it can be realized as a compactification of string or M-theory? In short: Does it belong to the landscape or the swampland of low energy actions [19]?

As we will demonstrate, the SO(32) heterotic string does provide us with a huge new class of quite promising vacua, which by S-duality significantly extends the set of open string models studied so far. Besides these more phenomenological questions, following the general spirit of the string landscape idea, it is desirable to compare and classify vacua in as different regions of the string moduli space as possible.

In [20, 21] we have given a general description of $E_8 \times E_8$ and SO(32) heterotic string compactifications on Calabi-Yau manifolds with general stable U(n) vector bundles. In particular, we have worked out the generalized Green-Schwarz mechanism involving not only the universal axion but also the axio-Kähler fields. In addition we have computed the perturbatively exact one-loop correction to the Donaldson-Uhlenbeck-Yau (DUY) equation, which turned out to be S-dual to the perturbative part of the Π-stability condition of B-type branes in type IIB string theory.

Let us emphasise that, on the one hand, for the $E_8 \times E_8$ string SU(n) bundles[1] are the most natural objects to work with; the point is that they give nice breaking patterns of E_8 down to some GUT gauge group and that for vanishing first Chern class of the bundle the one-loop corrected DUY equation is trivially satisfied. On the other hand for the SO(32) heterotic strings U(n) bundles play an analogous role for the same two reasons. First there are natural embeddings of U(n) into SO(32) and second for U(n) there is a chance to compensate the generically non-vanishing one-loop correction to the supersymmetry condition by the tree-level part. In [21] we mainly concentrated on the general framework of such SO(32) compactifications but only provided a small number of simple non-realistic examples. It is the aim of this paper to explore much more extensively the model building possibilities and to give a very concrete set of models.

To begin with we generalise the framework of [21] by including more general SO(32) breaking patterns which give rise to, for instance, chiral fields transforming in the symmetric

[1] For some earlier works of compactifications with U(n) bundles to four and six dimensions see also [22 – 26] and [27 – 30], respectively.

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representation of the unitary gauge groups. In addition, we also consider contributions from non-perturbative heterotic five-branes; in contrast to $E_8 \times E_8$ five-branes [31], they give rise to chiral matter and anomaly cancelling Green-Schwarz couplings.

Since for stability reasons we require supersymmetry at the string scale, a very explicit description of stable $\mathrm{U}(n)$ bundles over some Calabi-Yau manifolds is needed. We point out that by a twisting procedure one can define stable $\mathrm{U}(n)$ bundles on elliptically fibered Calabi-Yau's starting with the stable $\mathrm{SU}(n)$ bundles of Friedman, Morgan and Witten [7]. Even though in the course of this paper we work in the heterotic framework, the models we study can easily be S-dualized to compactifications of the type I string on smooth elliptically fibered Calabi-Yau's with stacks of D9-branes carrying non-trivial $\mathrm{U}(n)$ bundles as well as D5-branes[32].

Using this large class of bundles we search for Standard-like and GUT-like models, where the pattern for the chiral massless sectors is similar to the intersecting D-brane models studied earlier.

The remainder of this paper is organized as follows: section 2 gives the model building rules for SO(32) heterotic string compactifications on Calabi-Yau manifolds with products of $\mathrm{U}(n)$ bundles and additional five-branes. In section 3, we review the spectral cover construction for $\mathrm{SU}(n)$ bundles on elliptically fibered Calabi-Yau manifolds and introduce $\mathrm{U}(n)$ bundles by twisting with line bundles. For the base of the elliptic fibration chosen to be some del Pezzo surface, in section 4 we present a four-generation Pati-Salam type model and a three-generation MSSM like model. These are only two examples among a much larger class of models which our very restricted and preliminary search has produced. Section 5 contains our conclusions. Finally, some technical details on the Kähler cone and anomaly cancellation are collected in appendices A and B.

2. The SO(32) heterotic string with unitary bundles and five-branes

Following our recent work [21], in this section we review and extend a general class of compactifications of the SO(32) heterotic string involving direct sums of bundles with unitary structure groups. This also includes direct sums of abelian bundles.

2.1 Definition of a class of SO(32) heterotic string vacua

Concretely, we compactify the SO(32) heterotic string on a Calabi-Yau manifold X and consider decompositions of the gauge group SO(32) into

$$\mathrm{SO}(2M) \times \prod_{i=1}^{K} \mathrm{U}(M_i) \tag{2.1}$$

with $M + \sum_{i=1}^{K} M_i = 16$. To achieve such a breaking of the original SO(32) we give a background values to gauge field strength of vector bundles on the internal manifold with structure group $G = \prod_{i=1}^{K} \mathrm{U}(n_i)$, where each $\mathrm{U}(n_i)$ is diagonally embedded into a $\mathrm{U}(M_i)$ gauge factor with $M_i = n_i\, N_i$. Thus we consider bundles of the form

$$W = \bigoplus_{i=1}^{K} V_i \tag{2.2}$$

with V_i denoting a rank n_i unitary bundle. The resulting observable non-abelian gauge group is

$$H = \mathrm{SO}(2M) \times \prod_{i=1}^{K} \mathrm{U}(N_i)\,, \tag{2.3}$$

where as usual maximally only the anomaly-free part of the $\mathrm{U}(1)^K$ gauge factors remains in the low energy gauge group.

In addition to this perturbative sector we take into account the possible contribution from heterotic 5-branes (H5-branes), which can be understood as instantons of zero size [33 – 37]. For supersymmetry each H5-brane has to wrap an (in general reducible) holomorphic curve Γ on X. This means that the associated class $[\Gamma]$ is effective, i.e. lies inside the Mori cone of X. If Γ is irreducible, this really corresponds to a single H5-brane and gives rise to an additional Sp(2) gauge group in the effective action. Otherwise one decomposes Γ into the irreducible generators of the Mori cone Γ_a, $\Gamma = \sum_a N_a\,\Gamma_a$, $N_a \in \mathbb{Z}_0^+$. Due to the multiple wrapping around each irreducible curve Γ_a, the additional gauge group in the effective action gets enhanced to $\prod_a \mathrm{Sp}(2N_a)$. The decomposition into generators may not be unique and the gauge group may therefore vary in the different regions of the associated moduli space. However, its total rank and the total number of chiral degrees of freedom charged under the symplectic groups are only dependent on Γ, of course.

On the worldvolume of a SO(32) H5-brane lives a massless gauge field, which compared to the $E_8 \times E_8$ H5-brane leads to different low energy physics. Recall that in the latter case, the H5-brane supports an antisymmetric self-dual 2-form field and the rank of the gauge factor depends on the genus of the wrapped two-cycle [31].

In order to cancel the gravitational anomalies on the SO(32) H5-brane world-volume and by heterotic-type I duality, one can infer that the effective low energy action on the H5-branes has to have the usual Chern-Simons form

$$S_{H5_a} = -\mu_5 \int_{\mathbb{R}_{1,3}\times\Gamma_a} \sum_{n=0}^{1} B^{4n+2} \wedge \left(N_a + \frac{\ell_s^4}{4(2\pi)^2}\,\mathrm{tr}_{\mathrm{Sp}(2N_a)}\,F_a^2 \right) \wedge \frac{\sqrt{\hat{\mathcal{A}}(\mathrm{T}\Gamma_a)}}{\sqrt{\hat{\mathcal{A}}(\mathrm{N}\Gamma_a)}}, \tag{2.4}$$

with the H5-brane tension $\mu_5 = \frac{1}{(2\pi)^5\,(\alpha')^3}$. $\mathrm{T}\Gamma_a$ and $\mathrm{N}\Gamma_a$ denote the tangent bundle and the normal bundle, respectively, of the 2-cycle Γ_a, which for concreteness we take to be irreducible from now on and wrapped by a stack of N_a H5-branes. The curvature occurring in the definition of the $\hat{\mathcal{A}}$ genus is defined as $\mathcal{R} = -i\ell_s^2 R$ ($\ell_s \equiv 2\pi\sqrt{\alpha'}$). The sign of the Chern-Simons action is dictated by supersymmetry. With the choice in (2.4) the real part of the gauge kinetic function for the $\mathrm{Sp}(2N_a)$-group is indeed positive, as we demonstrate in appendix B. Note that (2.4) implies both the usual magnetic coupling to $B^{(6)}$ defined by $*_{10} dB^{(2)} = e^{2\phi_{10}} dB^{(6)}$ and a coupling to $B^{(2)}$, which is essential to cancel the mixed abelian-gravitational and abelian-symplectic anomalies by a generalized Green-Schwarz mechanism (see appendix B).

For our purpose it is useful to describe the H5-brane wrapping Γ_a as the skyscraper sheaf $\mathcal{O}|_{\Gamma_a}$ with the locally free resolution given by the Koszul sequence

$$0 \to \wedge^2 E_a^* \to E_a^* \to \mathcal{O}_X \to \mathcal{O}|_{\Gamma_a} \to 0\,. \tag{2.5}$$

reps.	$H=\prod_{i=1}^{K}\mathrm{SU}(N_i)\times \mathrm{U}(1)_i\times \mathrm{SO}(2M)\times\prod_{a=1}^{L}\mathrm{Sp}(2N_a)$
$(\mathrm{Adj}_{\mathrm{U}(N_i)})_{0(i)}$	$H^*(X, V_i\otimes V_i^*)$
$(\mathrm{Sym}_{\mathrm{U}(N_i)})_{2(i)}$	$H^*(X,\bigwedge^2 V_i)$
$(\mathrm{Anti}_{\mathrm{U}(N_i)})_{2(i)}$	$H^*(X,\bigotimes_s^2 V_i)$
$(\mathbf{N}_i,\mathbf{N}_j)_{1(i),1(j)}$	$H^*(X,V_i\otimes V_j)$
$(\mathbf{N}_i,\overline{\mathbf{N}}_j)_{1(i),-1(j)}$	$H^*(X,V_i\otimes V_j^*)$
$(\mathrm{Adj}_{\mathrm{SO}(2M)})$	$H^*(X,\mathcal{O})$
$(2\mathbf{M},\mathbf{N}_i)_{1(i)}$	$H^*(X,V_i)$
$(\mathrm{Anti}_{\mathrm{Sp}(2N_a)})$	$\mathrm{Ext}^*_X(\mathcal{O}\vert_{\Gamma_a},\mathcal{O}\vert_{\Gamma_a})$
$(\mathbf{N}_i,2\mathbf{N}_a)_{1(i)}$	$\mathrm{Ext}^*_X(V_i,\mathcal{O}\vert_{\Gamma_a})$
$(2\mathbf{N}_a,2\mathbf{N}_b)$	$\mathrm{Ext}^*_X(\mathcal{O}\vert_{\Gamma_a},\mathcal{O}\vert_{\Gamma_b})$

Table 1: Massless spectrum with the structure group taken to be $G=\prod_{i=1}^{K}\mathrm{U}(n_i)$.

Here E_a is a vector bundle of rank two and Γ_a is defined as the zero locus of a section of E_a. The Chern characters of this sheaf can be computed to be $\mathrm{ch}(\mathcal{O}|_{\Gamma_a})=(0,0,\gamma_a,0)$, where γ_a denotes the Poincaré dual four-form of the 2-cycle Γ_a. However, due to the overall minus sign in (2.4) relative to the gauge bundles on the Calabi-Yau, the five-brane Chern characters get an additional minus sign so that in the following $\mathrm{ch}(\mathcal{O}|_{\Gamma_a})=(0,0,-\gamma_a,0)$.

2.2 The massless spectrum

For the low energy effective theory it is important to know the massless four-dimensional spectrum. The perturbative spectrum can be determined from the decomposition of the adjoint representation of SO(32) into representations of $\prod_i \mathrm{U}(N_i)\times\mathrm{U}(n_i)$

$$\mathbf{496}\rightarrow\left(\begin{array}{c}(\mathrm{Anti}_{\mathrm{SO}(2M)})_0\\ \sum_{j=1}^{K}(\mathrm{Adj}_{\mathrm{U}(N_j)};\mathrm{Adj}_{\mathrm{U}(n_j)})\\ \sum_{j=1}^{K}(\mathrm{Anti}_{\mathrm{U}(N_j)};\mathrm{Sym}_{\mathrm{U}(n_j)})+(\mathrm{Sym}_{\mathrm{U}(N_j)};\mathrm{Anti}_{\mathrm{U}(n_j)})+\mathrm{h.c.}\\ \sum_{i<j}(\mathbf{N}_i,\mathbf{N}_j;\mathbf{n}_i,\mathbf{n}_j)+(\mathbf{N}_i,\overline{\mathbf{N}}_j,\mathbf{n}_i,\overline{\mathbf{n}}_j)+\mathrm{h.c.}\\ \sum_{j=1}^{K}(2M,\mathbf{N}_j;\mathbf{n}_j)+\mathrm{h.c.}\end{array}\right). \tag{2.6}$$

Due to the more general decomposition of SO(32), in contrast to [21], we now also get chiral matter in the symmetric representation of $\mathrm{U}(N_i)$. In addition new chiral matter appears from the non-perturbative H5-branes, which is absent for the H5-branes in $E_8\times E_8$ heterotic string compactifications [31]. In the latter case this is in accord with the possibility of moving the 5-branes into the eleven-dimensional bulk in the Horava-Witten theory. The resulting massless spectrum of the SO(32) string arising both in the perturbative and the non-perturbative sector is listed in table 1.

The matter arising in the H5-brane sector is described by appropriate extension groups. Following for instance [38], the global extension groups $\mathrm{Ext}^*_X(\mathcal{E},\mathcal{F})$ of two coherent sheaves on X give the sheaf theoretic generalisation of the cohomology groups $H^*(X,\mathcal{E}\otimes\mathcal{F}^*)$ for vector bundles on smooth manifolds. In particular, one can show that [38]

$$\mathrm{Ext}^1_X(\mathcal{O}|_{\Gamma_a},\mathcal{O}|_{\Gamma_a})=H^1(\Gamma_a,\mathcal{O})+H^0(\Gamma_a,\mathrm{N}\Gamma_a)\,, \tag{2.7}$$

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where the first term contains the possible Wilson line moduli on the H5-brane and the second term the geometric deformations of the two-cycles $\Gamma_a \subset X$. All these chiral supermultiplets transform in the antisymmetric representation of the symplectic gauge factor.

The perturbative chiral spectrum can be determined from the Euler characteristics of the various bundles U occurring in the decomposition of SO(32),

$$\chi(X,U) = \sum_{n=0}^{3}(-1)^n \dim H^n(X,U) = \int_X \left[\mathrm{ch}_3(U) + \frac{1}{12}\, c_2(T)\, c_1(U)\right]. \tag{2.8}$$

For more general coherent sheaves the righthand side in (2.8) measures the alternating sum of the dimensions of the global extensions. Note that in the non-perturbative sector, the H5-branes give rise to chiral matter in the bifundamental $(\mathbf{N}_i, 2\mathbf{N}_a)_{1(i)}$, which is counted by the index

$$\chi(X, V_i \otimes \mathcal{O}|^*_{\Gamma_a}) = -\int_X c_1(V_i) \wedge \gamma_a\,. \tag{2.9}$$

Again γ_a denotes the Poincaré dual 4-form corresponding to the 2-cycle Γ_a. Consistent with the absence of chiral matter for symplectic gauge groups only, for two H5-branes wrapping 2-cycles Γ_a and Γ_b one gets $\chi(X, \mathcal{O}|_{\Gamma_a} \otimes \mathcal{O}|^*_{\Gamma_b}) = 0$.

2.3 Model building constraints

Let us summarize the major model building constraints arising from supersymmetry and tadpole cancellation [21] including also H5-branes.

- The vector bundles have to be holomorphic and the non-holomorphic hermitean Yang-Mills equation is modified at one-loop. The perturbatively exact integrability condition reads

$$\frac{1}{2}\int_X J \wedge J \wedge c_1(V) - g_s^2\, \ell_s^4 \int_X \left(\mathrm{ch}_3(V) + \frac{1}{24}\, c_1(V)\, c_2(T)\right) = 0\,, \tag{2.10}$$

  where $g_s = e^{\phi_{10}}$ and $\ell_s = 2\pi\sqrt{\alpha'}$. Of course this equation has to hold inside the Kähler cone. Also the local supersymmetry variation of the gauge fields change accordingly and a unique supersymmetric solution exists if the bundle is π_h-stable (which is the S-dual of the perturbative part of type IIB Π-stability [39]). In fact, one can show that any μ-stable bundle is also π_h-stable [40]. Strictly speaking, this is true in the perturbative regime, i.e. for sufficiently large $r_i \gg 1$ and small $g_s \ll 1$. This is the regime in which we work anyway to suppress non-perturbative corrections.

  An additional constraint arises from the requirement that the one-loop corrected gauge couplings are real

$$\frac{n}{3!}\int_X J \wedge J \wedge J - g_s^2\, \ell_s^4 \int_X J \wedge \left(\mathrm{ch}_2(V) + \frac{n}{24}\, c_2(T)\right) > 0 \tag{2.11}$$

  in the perturbative regime, i.e. for $g_s \ll 1$ and $r_i \gg 1$. Note that there will be additional non-perturbative stringy and α' corrections to (2.10) and (2.11).

- The Bianchi identity for the three-form field strength H_3 is given by

$$dH_3 = \ell_s^2 \left(\frac{1}{4\,(2\pi)^2} \left[\mathrm{tr}\, R^2 - \mathrm{tr}\, F^2 \right] + \sum_a N_a\, \gamma_a \right) , \tag{2.12}$$

where the traces are taken in the fundamental representation of SO(1, 9) and SO(32), respectively. We have included the magnetic coupling of the H5-branes. For decompositions of the type (2.2), the resulting constraint from integrating the perturbative and H5-brane contributions over an internal 4-cycle reads

$$\sum_{i=1}^{K} N_i\, \mathrm{ch}_2(V_i) - \sum_{a=1}^{L} N_a \gamma_a = -c_2(T) \tag{2.13}$$

in cohomology.

- The absence of a global Witten anomaly on the H5-branes requires that the number of chiral fermions in the fundamental of the $\mathrm{Sp}(2N_a)$ groups be even [41]. In order to satisfy this constraint not only in the vacuum, but also in every topological sector of the theory, we insist that this holds even for every probe brane [42]. Inspection of (2.9) translates this into the constraint

$$c_1(W) = \sum_i N_i\, c_1(V_i) \in H^2(X, 2\mathbb{Z})\,. \tag{2.14}$$

Note that this condition was originally derived from the absence of anomalies in the two-dimensional non-linear sigma model [43, 44]. In type I string theory this is nothing else than the torsion K-theory constraint for the non-BPS D7-brane [42].

3. Construction of U(n) bundles

We consider elliptically fibered Calabi-Yau manifolds with a section and define SU(n) vector bundles à la Friedman, Morgan and Witten (FMW) via the spectral cover construction [7]. This construction has originally been designed for vector bundles with SU(n) structure group and has been generalized to U(n) bundles in [26]. Here we take a different approach and simply define U(n) bundles by twisting SU(n) bundles with line bundles. But first let us briefly review the spectral cover construction.

3.1 Elliptically fibered Calabi-Yau manifolds

An elliptically fibered Calabi-Yau threefold X consists of a complex two-dimensional base B and a projection

$$\pi : X \to B \tag{3.1}$$

with the property that the fiber over each point is an elliptic curve E_b. One requires that these bundles admit a section

$$\sigma : B \to X \tag{3.2}$$

which identifies the base as a submanifold of the space X. One assumes that the elliptic fibers are described by a Weierstrass equation,

$$zy^2 = 4x^3 - g_2 x z^2 - g_3 z^3 \,, \tag{3.3}$$

where g_2 and g_3 are sections of the line bundles $\mathcal{L}^4$ and $\mathcal{L}^6$ respectively. For X to be a Calabi-Yau manifold the canonical line bundle $\mathcal{K}_X$ has to be trivial, which in this case tells us that $\mathcal{L} = \mathcal{K}_B^{-1}$, i.e.

$$c_1(\mathcal{L}) = c_1(\mathcal{K}_B^{-1}) = c_1(B) \,. \tag{3.4}$$

If the base is smooth and preserves only $\mathcal{N} = 1$ supersymmetry in four dimensions, it is restricted to a del Pezzo surface, a Hirzebruch surface, an Enriques surface or a blow up of a Hirzebruch surface. In [7] it has been shown that the second Chern class of the tangent bundle can be expressed as

$$c_2(TX) = 12\sigma c_1(B) + c_2(B) + 11 c_1(B)^2 \,, \tag{3.5}$$

where σ denotes the two-form of the section, i.e. the Poincaré dual of the four cycle $B \subset X$. It has also been shown that $\sigma^2 = -\sigma c_1(B)$.

3.2 The spectral cover construction

A set of vector bundles with structure group SU(n) on X can be described by the so-called spectral cover construction. The idea is to use a simple description of SU(n) bundles over the elliptic fibers and then globally glue them together to define bundles over X. One of the many nice properties of such a construction is that eventually the Chern classes of such bundles can be computed entirely in terms of data defined on the base B.

The starting point is that on the elliptic fibers semi-stable SU(n) vector bundles split into direct sums of line bundles of degree zero. That means that each line bundle can be written as $\mathcal{O}(Q_i) \otimes \mathcal{O}(p)^{-1}$, where Q_i is a unique point on the elliptic fiber. Therefore, for an SU(n) bundle one obtains an n-tuple of such points fibered over the base of the elliptic fibration. This is the so-called spectral cover C, which is a ramified n-fold cover of B with $\pi_C : C \to B$. In order to specify the cohomology class $[C]$ of C in $H^2(X, \mathbb{Z})$, one has the freedom to twist by an additional line bundle η whose restriction to the elliptic fiber is of degree zero. For SU(n) bundles this restriction has to be the trivial line bundle on the fiber, which can be achieved by choosing for η the pull-back of a line bundle on the base. Therefore, the spectral cover lies in the cohomology class

$$[C] = n\sigma + \eta \tag{3.6}$$

with η being an effective class in B.

In order to specify a bundle V on X, besides the spectral cover C in addition one has to choose a line bundle $\mathcal{N}$ on C such that $V|_B = \pi_C {*} \mathcal{N}$, where the push-forward bundle $\pi_C {*} \mathcal{N}$ is a bundle on B whose fiber at a point b is the direct sum of n lines $\mathcal{N}|_{Q_i}$.

The entire vector bundle V on X is described by using the Poincaré bundle $\mathcal{P}$, which is a line bundle on the fiber product $Y = X \times_B C$ with

$$c_1(\mathcal{P}) = \Delta - \sigma_1 - \sigma_2 - c_1(B) \,. \tag{3.7}$$

Here Δ denotes the diagonal divisor in Y and σ_1 and σ_2 the sections $\sigma_1 : B \to X \subset Y$ and $\sigma_2 : B \to C \subset Y$. The vector bundle V is then defined as

$$V = \pi_{1*}(\pi_2^* \mathcal{N} \otimes \mathcal{P}) \,, \tag{3.8}$$

where π_1 and π_2 denote the two projections of the fiber product $X \times_B C$ onto the two factors X and C.

For SU(n) bundles the condition $c_1(V) = 0$ allows one to fix $c_1(\mathcal{N})$ in terms of $\sigma, \eta, c_1(B)$ and a rational number λ

$$c_1(V) = \pi_{C*}\left(c_1(\mathcal{N}) + \frac{1}{2}c_1(C) - \frac{1}{2}\pi_C^* c_1(B) - \gamma\right) \tag{3.9}$$

with $\gamma = \lambda(n\sigma - \pi_C^*\eta + n\pi_C^* c_1(B))$. The other two Chern classes $c_2(V)$ and $c_3(V)$ can be determined by applying the Grothendieck-Riemann-Roch (GRR) theorem

$$\pi_{1*}\left(\mathrm{ch}(\pi_2^* \mathcal{N} \otimes \mathcal{P})\, \mathrm{Td}(X \times_B C)\right) = \mathrm{ch(V)}\, \mathrm{Td}(X) \,, \tag{3.10}$$

where the push-forward π_{1*} of a form is defined as integration over the fiber. For completeness let us list the resulting Chern classes:

- The first Chern class vanishes
$$c_1(V) = 0 \,. \tag{3.11}$$
- The second Chern class can be expanded in a piece along the base and a piece along the fiber
$$c_2(V) = \eta\, \sigma + \omega \tag{3.12}$$
with ω defined as
$$\omega = -\frac{1}{24}\, c_1(B)^2\, (n^3 - n) + \frac{1}{2}\left(\lambda^2 - \frac{1}{4}\right) n\eta\, (\eta - nc_1(B)) \,. \tag{3.13}$$
- The third Chern class after integration over the fiber has the following simple form
$$c_3(V) = 2\, \lambda\eta(\eta - nc_1(B)) \tag{3.14}$$
and leads to the Euler characteristic
$$\chi(V) = \lambda\eta(\eta - nc_1(B)) \,. \tag{3.15}$$

Note that all classes can be expressed essentially by classes living on the base manifold B. These formulas are supplemented by additional relations for the defining quantities guaranteeing that $c_1(\mathcal{N})$ is an integer class:

$$n \text{ odd}: \ \lambda \in \mathbb{Z} + \frac{1}{2} \tag{3.16}$$

$$n \text{ even}: \lambda \in \mathbb{Z} \,, \qquad \eta + c_1(B) = 0 \mod 2 \tag{3.17}$$

$$\lambda \in \mathbb{Z} + \frac{1}{2} \,, \quad c_1(B) = 0 \mod 2 \,. \tag{3.18}$$

The bundles of this construction so far are μ-semi-stable. For them to be truly stable the spectral cover needs to be irreducible [45]. It has been shown in [46] that this boils down to two conditions on the curve η:

- The linear system $|\eta|$ has to be base point free.
- The curve $\eta - n\, c_1(B)$ has to be effective.

If this is the case then the $\mathrm{SU}(n)$ bundles are μ-stable.[2]

3.3 Twisting by a line bundle

One way to define stable $\mathrm{U}(n)$ bundles is via twisting with a line bundle on X. As before we start with a stable $\mathrm{SU}(n)$ bundle as it arises from the Fourier Mukai transform

$$V = \pi_{1*}(\pi_2^* \mathcal{N} \otimes \mathcal{P}) \tag{3.19}$$

of any line bundle $\mathcal{N}$ over the spectral cover C. In addition we take an arbitrary line bundle $\mathcal{Q}$ on X with

$$c_1(\mathcal{Q}) = q\,\sigma + c_1(\zeta) \tag{3.20}$$

and $c_1(\zeta) \in H^2(B, \mathbb{Z})$. Then we can define the twisted bundle

$$V_{\mathcal{Q}} = V \otimes \mathcal{Q}\,, \tag{3.21}$$

which has non-vanishing first Chern class unless $\mathcal{Q}$ is trivial. A bundle V is μ-stable if and only if $V \otimes \mathcal{Q}$ is stable for every line bundle $\mathcal{Q}$ [47, 48]. Therefore, all these twisted $\mathrm{U}(n)$ bundles are μ-stable if the $\mathrm{SU}(n)$ bundle is.

Using $\mathrm{ch}(V \otimes \mathcal{Q}) = \mathrm{ch}(V)\,\mathrm{ch}(\mathcal{Q})$ it is an easy exercise to compute the resulting Chern classes:

- The first Chern class is given by

$$c_1(V) = n\,q\,\sigma + n\,c_1(\zeta)\,. \tag{3.22}$$

- The second Chern character can be expanded in a piece along the base and a piece along the fiber

$$\mathrm{ch}_2(V) = \left[-\eta + \frac{n\,q}{2}\Big(2\,c_1(\zeta) - q\,c_1(B)\Big)\right]\sigma + a_F \tag{3.23}$$

with

$$a_F = \frac{n}{2} c_1(\zeta)^2 - \omega \tag{3.24}$$

and ω defined as usual

$$\omega = -\frac{1}{24}\,c_1(B)^2\,(n^3 - n) + \frac{1}{2}\left(\lambda^2 - \frac{1}{4}\right) n\eta\,(\eta - nc_1(B))\,. \tag{3.25}$$

[2] In fact, the proof of stability assumes that the Kähler parameter of the fiber lies in a certain range near the boundary of the Kähler cone, i.e. $J = \epsilon\sigma + J_B$ with sufficiently small ϵ [45]. Since the value of ϵ is not known, in all models involving the spectral cover constructions it is therefore a subtle issue if the region of stability overlaps with the perturbative regime, which is needed to have control over non-perturbative effects. We thank B. Andreas for drawing our attention to this issue.

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- The third Chern character after integration over the fiber has the following form

$$\begin{aligned} \mathrm{ch}_3(V) &= \lambda\eta(\eta - nc_1(B)) - \eta\, c_1(\zeta) + q\left(\frac{n}{2}\, c_1(\zeta)^2 - \omega\right) + \\ &\quad + q\, c_1(B)\left(\eta - \frac{n\, q}{2}\, c_1(\zeta) + \frac{n\, q^2}{6}\, c_1(B)\right), \end{aligned} \tag{3.26}$$

which leads to the Euler characteristic

$$\begin{aligned} \chi(V) &= \lambda\eta(\eta - nc_1(B)) - (\eta - nc_1(B))\, c_1(\zeta) + q\left(\frac{n}{2}\, c_1(\zeta)^2 - \omega\right) + \\ &\quad + q\, c_1(B)\left(\eta - \frac{n\, q}{2}\, c_1(\zeta) + \frac{n\, q^2}{6}\, c_1(B)\right) + \frac{n\, q}{12}\left(c_2(B) - c_1(B)^2\right). \end{aligned}$$

Such twisted unitary bundles will be the class of bundles we are going to use for our search of Standard-like and GUT models. The above construction also includes U(1) bundles, if V is simply the trivial line bundle. In the formulas for the Chern classes this boils down to setting $n = 1$ and $\eta = 0$.

In order to compute the Euler characteristics of products of bundles $V_i \otimes V_j$ appearing in table 1 one utilizes the formula

$$\chi(V_i \otimes V_j) = n_i\, \chi(V_j) + n_j\, \chi(V_i) + c_1(V_i)\, \mathrm{ch}_2(V_j) + \mathrm{ch}_2(V_i)\, c_1(V_j)\,. \tag{3.27}$$

Finally, for the Euler characteristic of the anti-symmetric product bundle $\bigwedge^2 V$ one obtains

$$\chi(\textstyle\bigwedge^2 V) = (n-4)\, \chi(V) + c_1(V)\left(\mathrm{ch}_2(V) + \frac{1}{4} c_2(T)\right) \tag{3.28}$$

and for the symmetric product bundle $\bigotimes_s^2 V$

$$\chi(\textstyle\bigotimes_s^2 V) = (n+4)\, \chi(V) + c_1(V)\left(\mathrm{ch}_2(V) - \frac{1}{4} c_2(T)\right). \tag{3.29}$$

3.4 The supersymmetry conditions

Now we have collected most of the ingredients to construct bundles and compute the chiral spectrum of SO(32) heterotic string compactifications on elliptically fibered Calabi-Yau threefolds. It only remains to evaluate the loop-corrected DUY condition. For the U(n) bundles we obtain

$$\begin{aligned} &\frac{n}{2}\, r_\sigma\left(2J_B - r_\sigma\, c_1(B)\right)\, (c_1(\zeta) - q\, c_1(B)) + \frac{nq}{2} J_B^2 = \\ &\quad = g_s^2\left[\chi(V) - \frac{n}{2} c_1(\zeta)\, c_1(B) - \frac{nq}{24}\left(c_2(B) - c_1(B)^2\right)\right] \end{aligned} \tag{3.30}$$

after expressing $J = \ell_s^2(r_\sigma\, \sigma + J_B)$ in terms of J_B, the Kähler form on the base B. This equation has to be satisfied inside the Kähler cone for the model to be well-defined. The constraints on the Kähler moduli resulting from this requirement are collected in appendix A.

The positivity condition on the real part of the gauge kinetic function for a U(N) factor leads to the second constraint

$$\begin{aligned}
&\frac{n}{3!} r_\sigma \left(r_\sigma^2 c_1(B)^2 - 3 r_\sigma c_1(B) J_B + 3 J_B^2\right) - \\
&- g_s^2 \left[(r_\sigma c_1(B) - J_B) \left(\eta - \frac{nq}{2}\left(2c_1(\zeta) - q c_1(B)\right)\right) + r_\sigma a_F \right] - \\
&- g_s^2 \frac{n}{2} \left[c_1(B) J_B + \frac{r_\sigma}{12}\left(c_2(B) - c_1(B)^2\right)\right] > 0\,. \qquad (3.31)
\end{aligned}$$

These conditions impose strong constraints on the bundles to be put simultaneously on the manifold X. In general each U(n) bundle freezes one combination of the dilaton and the $b_2(B)+1$ radii.

4. Semi-realistic models on del Pezzo surfaces

Let us now choose as the base manifold one of the del Pezzo surfaces dP_r with $r = 0, \ldots, 9$, which are defined by blowing up r points on $\mathbb{P}_2$. This means that $H^2(\mathrm{dP_r})$ is generated by the $r+1$ elements $l, E_1, \ldots, E_r$, where the class l is inherited from $\mathbb{P}_2$ and the E_m denote the r exceptional cycles introduced by the blow-ups. The intersection form can be computed as

$$l \cdot l = 1\,, \qquad l \cdot E_m = 0\,, \qquad E_m \cdot E_n = -\delta_{m,n}\,, \qquad (4.1)$$

and the Chern classes read

$$c_1(dP_r) = 3l - \sum_{m=1}^{r} E_m\,, \qquad c_2(dP_r) = 3 + r\,. \qquad (4.2)$$

For the second Chern class of X we obtain

$$c_2(TX) = 12\sigma c_1(B) + (102 - 10r)\, F\,. \qquad (4.3)$$

Now for a vector bundle V_i we can expand η_i and $c_1(\zeta_i)$ in a cohomological basis

$$\eta_i = \eta_i^{(0)}\, l + \sum_{m=1}^{r} \eta_i^{(m)}\, E_m\,, \qquad c_1(\zeta_i) = \zeta_i^{(0)}\, l + \sum_{m=1}^{r} \zeta_i^{(m)}\, E_m\,. \qquad (4.4)$$

As mentioned before we have to require that η is effective and that for stability $\eta - n\, c_1(B)$ is effective as well. Fortunately, all effective curves on dP_r have been classified in [49] and we list the reformulated result of [46] in table 2 for completeness.

Moreover, $|\eta|$ is known to be base point free if $\eta \cdot E \geq 0$ for every curve E with $E^2 = -1$ and $E \cdot c_1(B) = 1$. Such curves are precisely given by the generators of the Mori cone listed in table 2.

From our discussion it is clear that the parameter space of potentially consistent vacua is extremely huge and a completely systematic search for interesting models appears challenging. In the remainder of this section we present two semi-realistic examples which our very preliminary and restrictive survey has produced and whose properties are typical of a large set of solutions that can easily be generated. In fact, we have only covered a tiny fraction of the solution space of vector bundles on elliptic fibrations over dP_3 and dP_4.

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r	Generators	#
1	$E_1,\, l-E_1$	2
2	$E_i,\, l-E_1-E_2$	3
3	$E_i,\, l-E_i-E_j$	6
4	$E_i,\, l-E_i-E_j$	10
5	$E_i,\, l-E_i-E_j,\, 2l-E_1-E_2-E_3-E_4-E_5$	16
6	$E_i,\, l-E_i-E_j,\, 2l-E_i-E_j-E_k-E_l-E_m$	27
7	$E_i,\, l-E_i-E_j,\, 2l-E_i-E_j-E_k-E_l-E_m,$ $3l-2E_i-E_j-E_k-E_l-E_m-E_n-E_o$	56
8	$E_i,\, l-E_i-E_j,\, 2l-E_i-E_j-E_k-E_l-E_m,$ $3l-2E_i-E_j-E_k-E_l-E_m-E_n-E_o,$ $4l-2(E_i+E_j+E_k)-\sum_{i=1}^{5}E_{m_i},$ $5l-2\sum_{i=1}^{6}E_{m_i}-E_k-E_l,\, 6l-3E_i-2\sum_{i=1}^{7}E_{m_i}$	240
9	$f=3-\sum_{i=1}^{9}E_i$, and $\{y_a\}$ with $y_a^2=-1,\, y_a\cdot f=1$	∞

Table 2: Generators for the Mori cone of each dP_r, $r=1,\ldots,9$. All indices $i,j,\ldots\in\{1,\ldots,r\}$ in the table are distinct. The effective classes can be written as linear combinations of the generators with integer non-negative coefficients.

$U(n_i)$	λ_i	η_i	q_i	ζ_i
$U(1)_a$	0	0	0	$-2l+3E_2+3E_3$
$U(2)_b$	0	$11l-3E_1-5E_2-3E_3$	0	$-l+\sum_{m=1}^{3}E_m$
$U(2)_c$	0	$7l-3E_1-E_2-3E_3$	0	$-4l+4\sum_{m=1}^{3}E_m$

Table 3: Defining data for a $U(1)\times U(2)^2$ bundle.

4.1 A four-generation Pati-Salam model on dP_3

As a first example we choose the basis of the elliptic fibration to be the del Pezzo surface dP_3. Then we embed a bundle with structure group $U(1)\times U(2)^2$ into $U(4)^3$ yielding the observable group

$$H = U(4)\times U(2)^2\times SO(8)\,. \tag{4.5}$$

The data for the twisted bundles are given in table 3.

It can be checked explicitly that $\eta_b+c_1(B)$ and $\eta_c+c_1(B)$ have only even expansion coefficients as required for n even and $\lambda\in\mathbb{Z}$. Furthermore, η_b and η_c as well as

$$\eta_b-2c_1(B)=5l-E_1-3E_2-E_3\,,\qquad \eta_c-2c_1(B)=l-E_1+E_2-E_3 \tag{4.6}$$

are effective and the linear systems $|\eta_b|$, $|\eta_c|$ are base point free, i.e. all intersections with the basis of the Mori cone listed in table 2 are non-negative. Therefore, the constructed bundles are indeed μ-stable.

Finally, the tadpole

$$c_2(T)=12\left[3l-\sum_{m=1}^{3}E_m\right]\sigma+72 \tag{4.7}$$

$\mathrm{U}(4)_a \times \mathrm{U}(2)_b \times \mathrm{U}(2)_c$	mult.
$(\overline{\mathbf{4}}, \overline{\mathbf{2}}, \mathbf{1})_{-1,-1,0}$	2
$(\overline{\mathbf{4}}, \mathbf{2}, \mathbf{1})_{-1,1,0}$	2
$(\mathbf{4}, \mathbf{1}, \overline{\mathbf{2}})_{1,0,-1}$	2
$(\mathbf{4}, \mathbf{1}, \mathbf{2})_{1,0,1}$	2

Table 4: Chiral spectrum of a four generation Pati-Salam model on dP_3.

is cancelled without adding H5-branes due to

$$\begin{aligned} \mathrm{ch}_2(V_a) &= -7\,, \\ \mathrm{ch}_2(V_b) &= [-11l + 3E_1 + 5E_2 + 3E_3]\,\sigma + 8\,, \\ \mathrm{ch}_2(V_c) &= [-7l + 3E_1 + E_2 + 3E_3]\,\sigma - 30\,. \end{aligned} \tag{4.8}$$

The resulting chiral spectrum is displayed in table 4. Observe in particular that there is no chiral state charged under SO(8) due to $\chi(V_i) = 0$ and that there are no symmetric or antisymmetric chiral states since in addition $\zeta_i \cdot \mathrm{ch}_2(V_i) = \zeta_i \cdot c_2(T) = 0$ for all i.

The analysis of the chiral spectrum shows that all three U(1) factors are anomaly-free. However, the mass matrix (see appendix B) has rank two, and only the linear combination $4\,\mathrm{U}(1)_b - \mathrm{U}(1)_c$ remains massless.

The resulting DUY conditions are very simple in this configuration since all one-loop contributions cancel,

$$\begin{aligned} r_\sigma\,(3r_2 + 3r_3 + 2r_0) &= 0\,, \\ r_\sigma \left(r_0 + \sum_{m=1}^{3} r_m \right) &= 0\,, \end{aligned}$$

and the positivity of the gauge kinetic functions requires

$$\begin{aligned} r_\sigma \left(2r_\sigma^2 - r_\sigma(3r_0 + \sum_{m=1}^{3} r_m) + r_0^2 - \sum_{m=1}^{3} r_m^2 \right) - g_s^2 \left(-14r_\sigma + 3r_0 + \sum_{m=1}^{3} r_m \right) &> 0\,, \\ r_\sigma \left(2r_\sigma^2 - r_\sigma(3r_0 + \sum_{m=1}^{3} r_m) + r_0^2 - \sum_{m=1}^{3} r_m^2 \right) - g_s^2\,(30r_\sigma - 8r_0 - 2r_1 - 4r_2 - 2r_3) &> 0\,, \\ r_\sigma \left(2r_\sigma^2 - r_\sigma(3r_0 + \sum_{m=1}^{3} r_m) + r_0^2 - \sum_{m=1}^{3} r_m^2 \right) + g_s^2\,(16r_\sigma + 4r_0 + 2r_1 + 2r_3) &> 0\,. \end{aligned}$$

These conditions can be fulfilled in the perturbative regime inside the Kähler cone, e.g. for arbitrary r_σ and $g_s < 0.16\,r_\sigma$, $r_0 = 1.8\,r_\sigma$, $r_1 = r_2 = r_3 = -0.6\,r_\sigma$.

4.2 A three-generation Standard-like model on dP_4

This section is devoted to a three-generation Standard-like model involving four vector bundles, where we now take the base manifold to be dP_4. It can be regarded as the generalized S-dual version of the four-stack models which have become popular in the

$\mathrm{U}(n_i)$	λ_i	η_i	q_i	ζ_i
$\mathrm{U}(1)_a$	0	0	1	$5l-3E_1-5E_2-E_3$
$\mathrm{U}(1)_b$	0	0	1	$-3l+5E_1+2E_2-E_3+E_4$
$\mathrm{SU}(2)_c$	0	$7l-3E_1-3E_2-E_3-E_4$	0	0
$\mathrm{U}(1)_d$	0	0	- 1	$-5l+3E_1+5E_2+E_3$

Table 5: Defining data for a $\mathrm{U}(1)\times\mathrm{U}(1)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ bundle.

framework of intersecting branes. Our aim is therefore to obtain a visible gauge group $\mathrm{U}(3)_a\times\mathrm{U}(2)_b\times\mathrm{U}(1)_c\times\mathrm{U}(1)_d$ and realize the quarks and leptons as appropriate bifundamentals. A possible choice of the hypercharge as a (massless) combination of the abelian factors is given by $Q_Y=\frac{1}{6}Q_a+\frac{1}{2}(Q_c+Q_d)$. In this case, also some of the (anti-)symmetric representations carry MSSM quantum numbers . The details of the chiral MSSM spectrum we try to reproduce can be found in table 6.

Among the many possibilities we consider the simple embedding of the structure group $G=\mathrm{U}(1)\times\mathrm{U}(1)\times\mathrm{U}(2)\times\mathrm{U}(1)$ into $\mathrm{U}(3)\times\mathrm{U}(2)\times\mathrm{U}(2)\times\mathrm{U}(1)$. This leads to

$$H=\mathrm{U}(3)\times\mathrm{U}(2)\times\mathrm{U}(1)\times\mathrm{U}(1)\times\mathrm{SO}(16) \tag{4.9}$$

modulo the issue of anomalous abelian factors. We choose the bundles characterized in table 5.

Note that V_c actually has structure group SU(2) rather than U(2) since its first Chern class vanishes, which however makes no difference in the group theoretic decomposition of SO(32). Again, one may verify explicitly that the conditions for μ-stability are satisfied. Let us also point out that the requirement (2.14) of cancellation of the Witten anomaly, which is non-trivial for odd N_a, is satisfied by the configuration. Furthermore, the $\mathrm{U}(1)_Y$ hypercharge is indeed massless as desired (see appendix B). However, since the rank of the mass matrix is two, we get another massless U(1) in the four-dimensional gauge group, which is readily identified as $\mathrm{U}(1)_c$. The perturbative low energy gauge group is therefore

$$H=\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)_Y\times\mathrm{U}(1)'\times\mathrm{SO}(16)\,. \tag{4.10}$$

The degeneracy of the bundle V_a and $V_d=V_a^*$ leads to a gauge enhancement of the $\mathrm{U}(3)_a$ and the $\mathrm{U}(1)_d$ to a U(4). Apart from these drawbacks, the configuration indeed gives rise to three families of the MSSM chiral spectrum as listed in table 6.

In addition, we get some chiral exotic matter in the antisymmetric of the U(2) and in the bifundamental of the SO(16) with the U(3) and U(2), respectively (see table 7).

In contrast to the previous example, the chosen bundles alone do not satisfy the tadpole cancellation condition. However, the resulting tadpole can be cancelled by including H5-branes, which demonstrates the importance of allowing for these non-perturbative objects. From the general form of the tadpole equation we find the four-form characterizing this tadpole to be

$$[W]=c_2(T)+\sum_{i=1}^{4}N_i\,\mathrm{ch}_2(V_i)=22\,F+(34l-8E_1-22E_2-14E_3-6E_4)\,\sigma\,. \tag{4.11}$$

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$\mathrm{U}(3)_a \times \mathrm{U}(2)_b \times \mathrm{U}(1)_c \times \mathrm{U}(1)_d \times \mathrm{SO}(16) \times \prod_a \mathrm{Sp}(2N_a)$				
MSSM particle	repr.	index	mult.	total
Q_L	$(\mathbf{3},\overline{\mathbf{2}};1,1)_{(1,-1,0,0)}$	$\chi(X, V_a \otimes V_b^*)$	8	
Q_L	$(\mathbf{3},\mathbf{2};1,1)_{(1,1,0,0)}$	$\chi(X, V_a \otimes V_b)$	-11	-3
u_R	$(\overline{\mathbf{3}},1;1,1)_{(-1,0,-1,0)}$	$\chi(X, V_a^* \otimes V_c^*)$	-3	
u_R	$(\overline{\mathbf{3}},1;1,1)_{(-1,0,0,-1)}$	$\chi(X, V_a^* \otimes V_d^*)$	0	-3
d_R	$(\overline{\mathbf{3}},1;1,1)_{(-1,0,1,0)}$	$\chi(X, V_a^* \otimes V_c)$	-3	
d_R	$(\overline{\mathbf{3}},1;1,1)_{(-1,0,0,1)}$	$\chi(X, V_a^* \otimes V_d)$	45	
d_R	$(\overline{\mathbf{3}}_A,1;1,1)_{(2,0,0,0)}$	$\chi(X, \bigotimes_s^2 V_a)$	-45	-3
L	$(1,\mathbf{2};1,1)_{(0,1,-1,0)}$	$\chi(X, V_b \otimes V_c^*)$	-7	
L	$(1,\mathbf{2};1,1)_{(0,1,0,-1)}$	$\chi(X, V_b \otimes V_d^*)$	-11	
L	$(1,\overline{\mathbf{2}};1,1)_{(0,-1,-1,0)}$	$\chi(X, V_b^* \otimes V_c^*)$	7	
L	$(1,\overline{\mathbf{2}};1,1)_{(0,-1,0,-1)}$	$\chi(X, V_b^* \otimes V_d^*)$	8	-3
e_R	$(1,1;1,1)_{(0,0,2,0)}$	$\chi(X, \bigwedge^2 V_c)$	0	
e_R	$(1,1;1,1)_{(0,0,0,2)}$	$\chi(X, \bigwedge^2 V_d)$	0	
e_R	$(1,1;1,1)_{(0,0,1,1)}$	$\chi(X, V_c \otimes V_d)$	-3	-3
ν_R	$(1,1;1,1)_{(0,0,-1,1)}$	$\chi(X, V_c^* \otimes V_d)$	-3	-3

Table 6: Chiral MSSM spectrum for a four-stack model with $Q_Y = \frac{1}{6}Q_a + \frac{1}{2}(Q_c + Q_d)$.

$\mathrm{U}(3)_a \times \mathrm{U}(2)_b \times \mathrm{U}(1)_c \times \mathrm{U}(1)_d \times \mathrm{SO}(16) \times \prod_a \mathrm{Sp}(2N_a)$				
MSSM particle	repr.	index	mult.	total
-	$(1,\mathbf{1}_A;1,1)_{(0,2,0,0)}$	$\chi(X, \bigotimes_s^2 V_b)$	-77	-77
-	$(\mathbf{3},1;\mathbf{16},1)_{(1,0,0,0)}$	$\chi(X, V_a)$	-1	-1
-	$(1,\mathbf{2};\mathbf{16},1)_{(0,1,0,0)}$	$\chi(X, V_b)$	-11	-11
-	$(1,1;\mathbf{16},1)_{(0,0,1,0)}$	$\chi(X, V_c)$	0	0
-	$(1,1;\mathbf{16},1)_{(0,0,0,1)}$	$\chi(X, V_d)$	1	1
-	$\sum_a(\mathbf{3},1;1,\mathbf{2N_a})_{(1,0,0,0)}$	$\chi(X, V_a \otimes \mathcal{O}\|_\Gamma)$	8	8
-	$\sum_a(1,\mathbf{2};1,\mathbf{2N_a})_{(0,1,0,0)}$	$\chi(X, V_b \otimes \mathcal{O}\|_\Gamma)$	56	56
-	$\sum_a(1,1;1,\mathbf{2N_a})_{(0,0,1,0)}$	$\chi(X, V_c \otimes \mathcal{O}\|_\Gamma)$	0	0
-	$\sum_a(1,1;1,\mathbf{2N_a})_{(0,0,0,1)}$	$\chi(X, V_d \otimes \mathcal{O}\|_\Gamma)$	-8	-8

Table 7: Chiral exotic spectrum for the four-stack model with $Q_Y = \frac{1}{6}Q_a + \frac{1}{2}(Q_c + Q_d)$. In the second column, the first two entries refer to the U(3) and U(2) factors, the third to the SO(16) group and the fourth collectively represents the symplectic charges. The U(1) charges are read off from the lower-case entries.

Its Poincaré dual class $[\Gamma] = 22\sigma + 34l - 8E_1 - 22E_2 - 14E_3 - 6E_4$ lies inside the Mori cone, i.e. is effective, and can thus be regarded as the homology class associated to a (reducible) holomorphic curve around which we may wrap a system of H5-branes. To determine the detailed spectrum and gauge group supported by the branes we must choose a decomposition of $[\Gamma]$ into irreducible effective classes around each of which we can wrap one H5-brane. These are given precisely by the generators of the Mori cone in table 2. Note that the decomposition is not unique and constitutes (part of) the moduli space of our model; what is universal is the total number of chiral degrees of freedom charged under the symplectic sector (see table 7) and its total rank. In our case, the latter is easily found to be 74. For instance, the decomposition

$$[\Gamma] = 22\,\sigma + 22(l - E_2 - E_3) + 12(l - E_1 - E_4) + 4E_1 + 8E_3 + 6E_4 \tag{4.12}$$

results in the symplectic gauge group $\mathrm{Sp}(44) \times \mathrm{Sp}(44) \times \mathrm{Sp}(24) \times \mathrm{Sp}(8) \times \mathrm{Sp}(16) \times \mathrm{Sp}(12)$. The bifundamental exotics between the MSSM group and this symplectic gauge sector can be determined with the help of (2.9). Ideally, this group would be hidden, of course.

The only independent DUY equations are those for V_a and V_b

$$\frac{1}{2}\left(r_0^2 - \sum_{m=1}^{4} r_m^2\right) + r_\sigma\left(2r_0 + 2r_1 + 4r_2 - r_4 - \frac{1}{2}r_\sigma\right) = -\frac{49}{12}g_s^2\,, \tag{4.13}$$

$$\frac{1}{2}\left(r_0^2 - \sum_{m=1}^{4} r_m^2\right) + r_\sigma\left(-6r_0 - 6r_1 - 3r_2 - 2r_4 + \frac{7}{2}r_\sigma\right) = -\frac{121}{12}g_s^2\,, \tag{4.14}$$

and only fix two of the Kähler moduli. This can well be achieved, together with positivity of the real part of the various gauge kinetic functions, inside the Kähler cone (see appendix A) and in the perturbative regime. E.g. by taking $r_2 = -2.5\,r_\sigma$, $r_3 = -1.1\,r_\sigma$, $r_4 = -\,r_\sigma$ and $g_s < 0.58\,r_\sigma$ for arbitrary r_σ, the solution for r_0 and r_1 satisfies all Kähler cone constraints. We can therefore always choose r_σ and g_s such that the model is indeed in the perturbative regime.

5. Conclusions

In this paper we have continued the systematic analysis of compactifications of the SO(32) heterotic string on Calabi-Yau manifolds.[3] Generalizing our earlier work [21] we have allowed for more general embeddings of the $\mathrm{U}(n)$ structure groups giving rise to a reduction of the rank of the gauge group as well as to chiral fermions transforming in the symmetric representation of the observable unitary gauge groups. In addition we have also included heterotic 5-branes, which contribute both to the 6-form tadpole and, unlike in the $E_8 \times E_8$ heterotic string, to the chiral massless spectrum.

As we demonstrate in [32], S-duality relates these heterotic models to type I compactifications, where of course the orders of terms in string and sigma model perturbation theory are different. The topological sector, however, is in complete agreement.

It is clear that this framework is extremely rich as long as one has control of stable vector bundles on certain Calabi-Yau spaces. Fortunately, on elliptically fibered Calabi-Yau manifolds the spectral cover construction of FMW provides a well controllable set of such stable vector bundles. In this paper we have considered stable $\mathrm{U}(n)$ bundles on elliptically fibered Calabi-Yau manifolds defined via twisting $\mathrm{SU}(n)$ bundles à la FMW by line bundles.

[3] In a recent paper [50] it was pointed out that, in constrast to the $E_8 \times E_8$ heterotic string, the SO(32) heterotic string theory might also contain non-perturbative open strings.

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We have only started a systematic search for Standard- respectively GUT-like models on elliptically fibered Calabi-Yau manifolds, choosing del Pezzo surfaces for the base. This paper reports on the very first results of such a search. Clearly, the models presented are not yet completely satisfactory, but they demonstrate that not only the $E_8 \times E_8$ heterotic string but also the long neglected SO(32) heterotic string can naturally give rise to Standard model-like string compactifications. The parameter space of consistent models is so huge that we have restricted our search to only a very tiny subset of possible solutions, in which we have nonetheless found various additional three-generation MSSM-like models similar to the one presented in section 4.2. Compared to the tremendous effort that has gone into the study of toroidal orientifold models and their by far smaller space of supersymmetric solutions we consider this result quite promising, both from the point of view of concrete model building and with regard to classification attempts in the context of the (open) string landscape. A more thorough and systematic search might be carried out in the context of a future StringVacuumProject (SVP).

A nice feature of a certain class of SU(n) bundles on Calabi-Yau manifolds is that also the string world-sheet theory is known to be well behaved and that the vacuum is not destabilised by world-sheet instanton corrections. This is not so clear for the class of models discussed here where the left-moving $\mathrm{U}(1)_L$ symmetry on the world-sheet is anomalous. More work is needed to clarify such more complicated issues.

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A. Kähler cone constraints on Calabi-Yau's with base dP$_r$

The DUY equations have to admit solutions for the Kähler parameters inside the Kähler cone, i.e. such that the integral of powers of the Kähler form over all appropriate cycles are positive,

$$\int_{2-\text{cycle}} J > 0\,, \qquad \int_{4-\text{cycle}} J^2 > 0\,, \qquad \int_X J^3 > 0\,. \tag{A.1}$$

We expand the Kähler form on the elliptically fibered Calabi-Yau as $J = l_s^2\,(r_\sigma\,\sigma + J_B)$ with $J_B = r_0\,l + \sum_{m=1}^r r_m E_m$ being the Kähler form on the base manifold dP$_r$ in terms of the canonical basis.

From the first constraint we read immediately that the radii must satisfy

$$r_\sigma > 0\,, \qquad r_0 > 0\,, \qquad r_m < 0 \qquad \text{for } m \in \{1, \dots, r\}\,. \tag{A.2}$$

The second inequality, $\int J^2 > 0$, holds precisely if in addition

$$r_0^2 - \sum_{m=1}^r r_m^2 > 0\,, \qquad r_\sigma < \frac{2}{3}\,r_0\,, \qquad r_\sigma < -2r_m \qquad \text{for } m \in \{1, \dots, r\}\,. \tag{A.3}$$

Finally positivity of the volume of the Calabi-Yau necessitates that also

$$r_\sigma^3\,(9-r) - 3r_\sigma^2\left(3r_0 + \sum_{m=1}^{r} r_m\right) + 3r_\sigma\left(r_0^2 - \sum_{m=1}^{r} r_m^2\right) > 0\,. \tag{A.4}$$

B. Details of the generalized Green-Schwarz mechanism

The trace identities of [21] are generalized for $\mathrm{U}(n_i)$ factors diagonally embedded into $\mathrm{U}(n_iN_i)$ to

$$\begin{aligned}
\mathrm{tr}\,F\overline{F}^3 &= 12\sum_{j=1}^{K} N_j\, f_j \wedge \left(4\,\mathrm{tr}_{\mathrm{U}(n_j)}\,\overline{F}^3 + \mathrm{tr}_{\mathrm{U}(n_j)}\,\overline{F}\sum_{i=1}^{K} N_i\,\mathrm{tr}_{\mathrm{U}(n_i)}\,\overline{F}^2\right),\\
\mathrm{tr}\,F^2\overline{F}^2 &= 4\sum_{j=1}^{K}\left(\mathrm{tr}_{\mathrm{SU}(N_j)}\,F^2 + N_j\,(f_j)^2\right)\wedge\left(12\;\mathrm{tr}_{\mathrm{U}(n_j)}\,\overline{F}^2 + n_j\sum_{i=1}^{K} N_i\;\mathrm{tr}_{\mathrm{U}(n_i)}\,\overline{F}^2\right) +\\
&\qquad + 8\sum_{i,j=1}^{K} N_iN_j\, f_i\, f_j \wedge \mathrm{tr}_{\mathrm{U}(n_i)}\,\overline{F}\;\mathrm{tr}_{\mathrm{U}(n_j)}\,\overline{F} + 2\;\mathrm{tr}_{\mathrm{SO}(2M)}\,F^2 \wedge \sum_{j=1}^{K} N_j\;\mathrm{tr}_{\mathrm{U}(n_j)}\,\overline{F}^2\,,\\
\mathrm{tr}\,F^2 &= 30\;\mathrm{tr}_{\mathrm{SO}(2M)}\,F^2 + 60\sum_{j=1}^{K} n_j\left(\mathrm{tr}_{\mathrm{SU}(N_j)}\,F^2 + N_j(f_j)^2\right),\\
\mathrm{tr}\,F\overline{F} &= 60\sum_{j=1}^{K} N_j f_j \wedge \mathrm{tr}_{\mathrm{U}(n_j)}\,\overline{F}\,,\\
\mathrm{tr}\,\overline{F}^2 &= 60\sum_{j=1}^{K} N_j\;\mathrm{tr}_{\mathrm{U}(n_j)}\,\overline{F}^2\,.
\end{aligned} \tag{B.1}$$

For details of the notation see section 3 and appendix B in [21].

With these relations, the mass terms for the axions are given by

$$\begin{aligned}
S^{\text{1-loop}}_{\text{mass}} &= \frac{1}{6(2\pi)^5\alpha'}\sum_{i=1}^{K} N_i \int_{\mathbb{R}_{1,3}} b_0^{(2)} \wedge f_i \int_X \left(\mathrm{tr}_{\mathrm{U}(n_i)}\,\overline{F}^3 - \frac{1}{16}\,\mathrm{tr}_{\mathrm{U}(n_i)}\,\overline{F}\wedge\mathrm{tr}\,\overline{R}^2\right),\\
S^{\text{tree}}_{\text{mass}} &= \frac{1}{(2\pi)^2\alpha'}\sum_{i=1}^{K}\sum_{k=1}^{h_{11}} N_i \int_{\mathbb{R}_{1,3}} (b_k^{(2)} \wedge f_i)\,[\mathrm{tr}_{\mathrm{U}(n_i)}\,\overline{F}]_k\,,
\end{aligned} \tag{B.2}$$

and the vertex couplings to the gauge fields and gravitons are given by

$$\begin{aligned}
S^{\text{1-loop}}_{\text{vertex}} &= \frac{1}{2\pi}\sum_{k=1}^{h_{11}} \int_{\mathbb{R}_{1,3}} b_k^{(0)} \wedge \left\{\sum_{i=1}^{K}\left(\mathrm{tr}_{\mathrm{SU}(N_i)}\,F^2 + N_i(f_i)^2\right)\left[\frac{1}{4}\;\mathrm{tr}_{\mathrm{U}(n_i)}\,\overline{F}^2 - \frac{n_i}{192}\,\mathrm{tr}\,\overline{R}^2\right]_k -\right.\\
&\qquad - \frac{1}{384}\,\mathrm{tr}_{\mathrm{SO}(2M)}\,F^2\left[\mathrm{tr}\,\overline{R}^2\right]_k +\\
&\qquad \left. + \frac{1}{768}\,\mathrm{tr}\,R^2\left[\mathrm{tr}\,\overline{R}^2 - 4\sum_{i=1}^{K} N_i\,\mathrm{tr}_{\mathrm{U}(n_i)}\,\overline{F}^2\right]_k\right\},\\
S^{\text{tree}}_{\text{vertex}} &= \frac{1}{8\pi}\int_{\mathbb{R}_{1,3}} b_0^{(0)} \wedge \left(2\sum_{i=1}^{K} n_i\left(\mathrm{tr}_{\mathrm{SU}(N_i)}\,F^2 + N_i(f_i)^2\right) + \mathrm{tr}_{\mathrm{SO}(2M)}\,F^2 - \mathrm{tr}\,R^2\right).
\end{aligned}$$

In addition, the H5-brane action (2.4) contains the vertex couplings

$$S_{\text{vertex}}^{H5} = -\frac{1}{8\pi}\sum_{k=1}^{h_{11}}\int_{\mathbb{R}_{1,3}}[\Gamma_a]_k\, b_k^{(0)} \wedge \left(\mathrm{tr}_{\mathrm{Sp}(2N_a)}\, F_a^2 - \frac{N_a}{24}\,\mathrm{tr}\, R^2\right) \tag{B.3}$$

with $[\Gamma_a]_k = \int_{\Gamma_a}\omega_k$.

One may verify that these couplings precisely cancel all abelian anomalies, in the case of the mixed abelian-gravitational ones only upon tadpole cancellation.

The mass terms can be rewritten as

$$S_{\text{mass}} = \sum_{i=1}^{K}\sum_{k=0}^{h_{11}}\frac{M_k^i}{2\pi\alpha'}\int_{\mathbb{R}_{1,3}} f_i \wedge b_k^{(2)} \tag{B.4}$$

with

$$M_k^i = \frac{N_i}{2\pi}\times\begin{cases}\chi(V_i) - \dfrac{1}{24}\displaystyle\int_X c_1(V_i)c_2(T) & \text{for } k=0\\ [c_1(V_i)]_k & \text{for } k\in\{1,\ldots,h_{11}\},\end{cases} \tag{B.5}$$

where $[c_1(V_i)]_k$ is the expansion coefficient of the k^{th} basic two form. The number of massive U(1) factors is then given by rank(M).

The gauge kinetic functions for the unitary factors are easily read off from the vertex couplings

$$\begin{aligned}\mathrm{f}_{\mathrm{SU}(N_i)} &= 2n_i\, S + \sum_{k=1}^{h_{11}} T_k\,\left(\mathrm{tr}_{\mathrm{U}(n_i)}(\overline{F}^2)_k - \frac{n_i}{48}(\mathrm{tr}\,\overline{R}^2)_k\right),\\ \mathrm{f}_{\mathrm{U}(1)_i} &= N_i\mathrm{f}_{\mathrm{SU}(N_i)}\end{aligned} \tag{B.6}$$

generalising the result in [21] to arbitrary n_i. In addition, the gauge kinetic functions for the symplectic factors are

$$\mathrm{f}_{\mathrm{Sp}(2N_a)} = \frac{1}{2\pi\ell_s^2}\int_{\Gamma_a}(J - iB)\,. \tag{B.7}$$

For the elliptic fibrations over del Pezzo surfaces discussed in this article, we obtain

$$\begin{pmatrix} M_0^i \\ M_\sigma^i \\ M_l^i \\ M_{m:m=1,\ldots,r}^i\end{pmatrix} = \frac{N_i}{2\pi}\times\begin{pmatrix}\chi(V_i) + n_i\left(q_i\left(\frac{1}{4}-\frac{r}{12}\right) - \frac{1}{2}\left(3\zeta_i^{(0)} + \sum_{m=1}^r \zeta_i^{(m)}\right)\right)\\ n_i q_i \\ n_i\zeta_i^{(0)} \\ n_i\zeta_i^{(m)}\end{pmatrix}. \tag{B.8}$$

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